ME-446: Liquid-gas interfacial heat and mass transfer

Boiling I

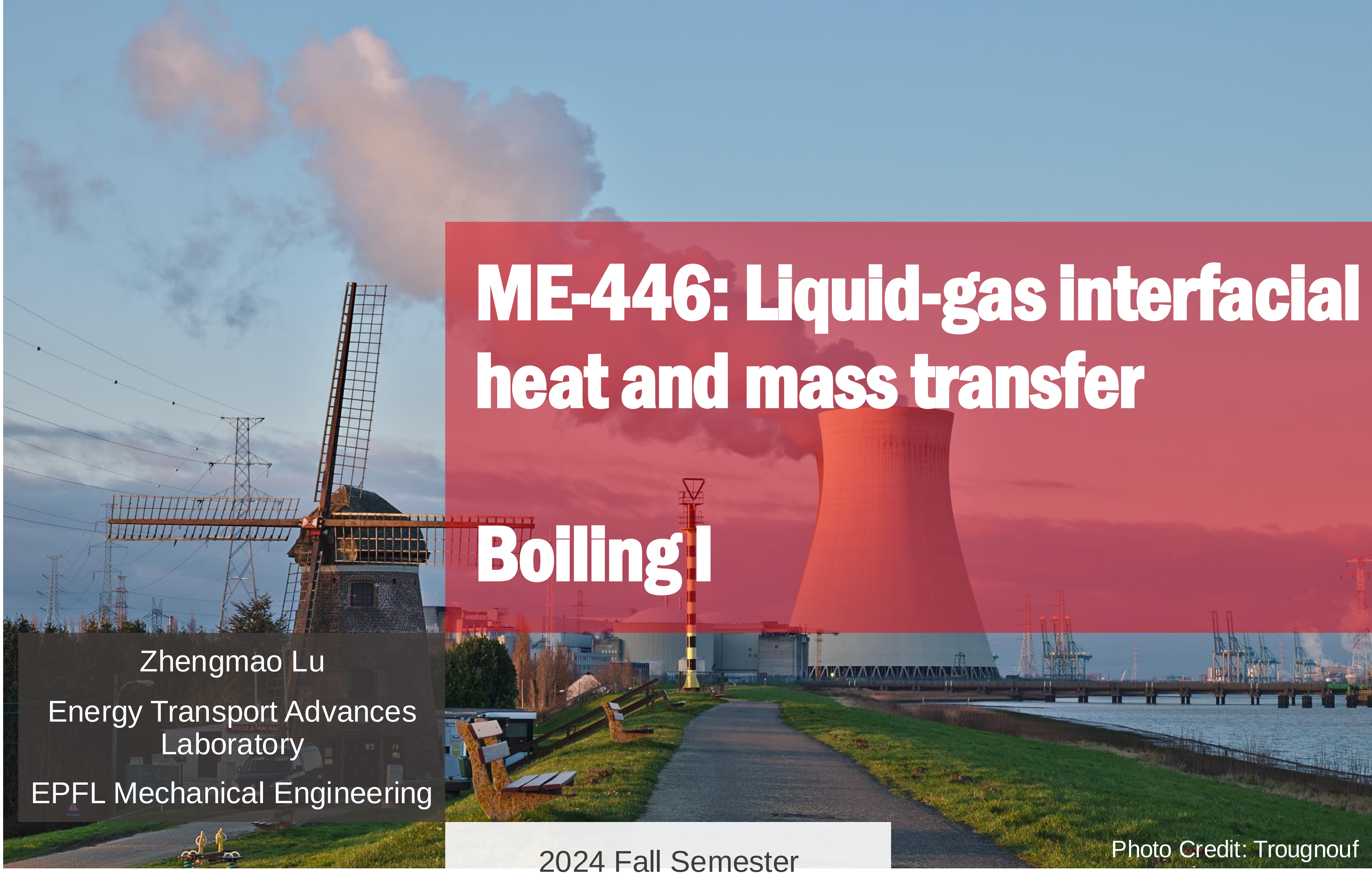

Photo Credit: Trougnouf

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Energy Transport Advances Laboratory

EPFL Mechanical Engineering

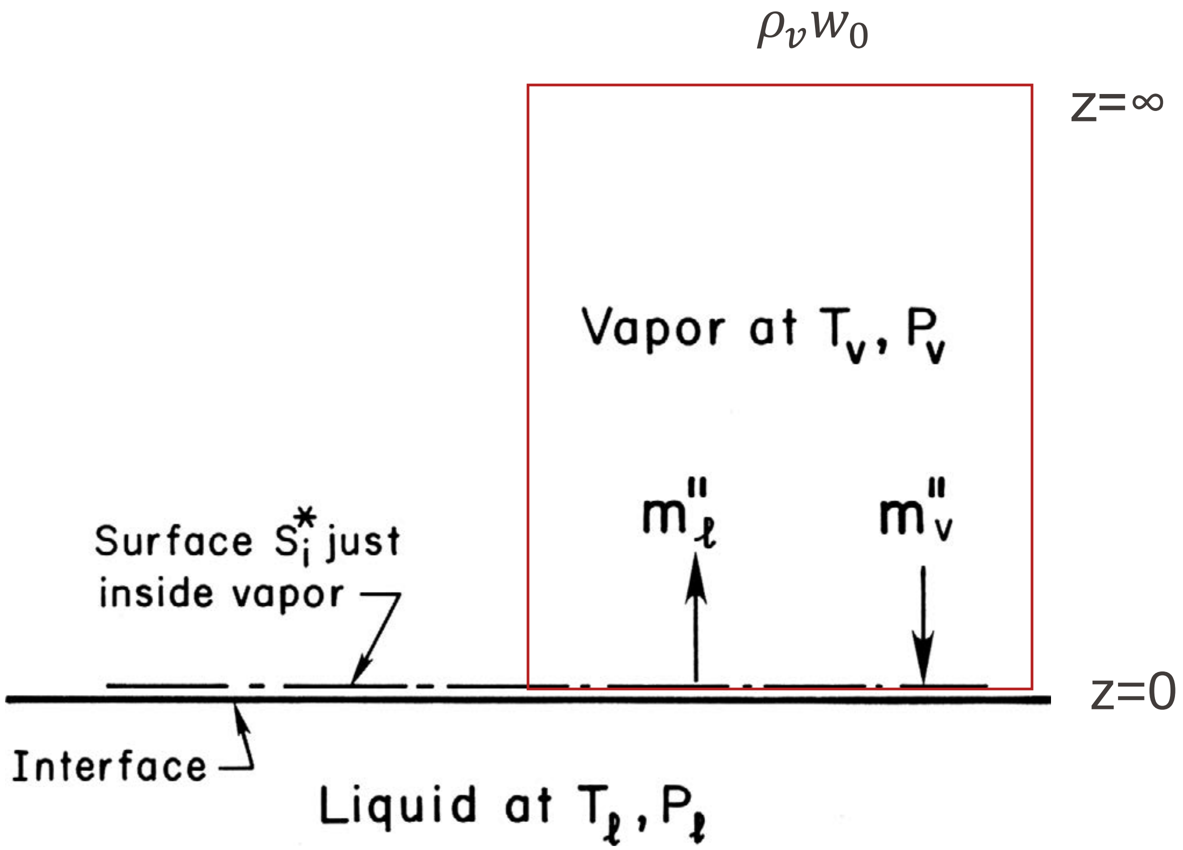
ETA-Lab
Energy Transport Advances

2024 Fall Semester

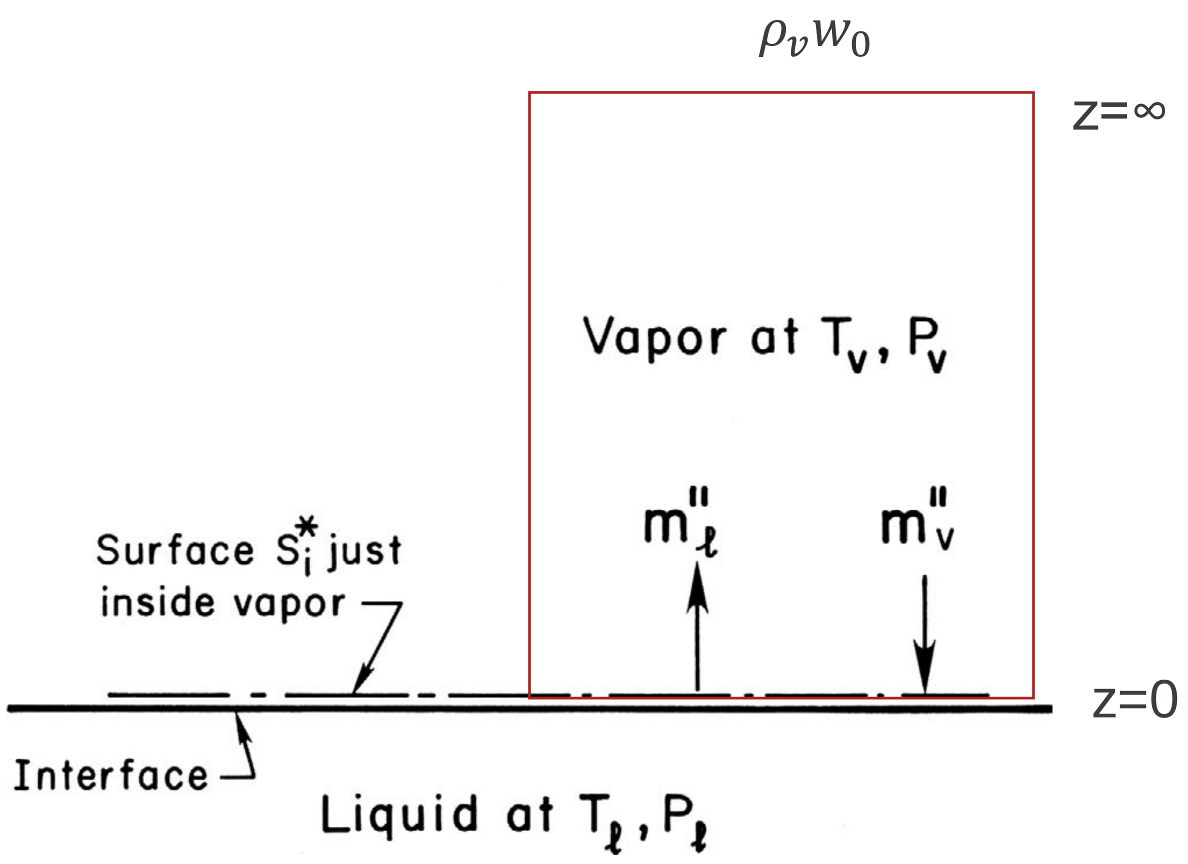
Last Time

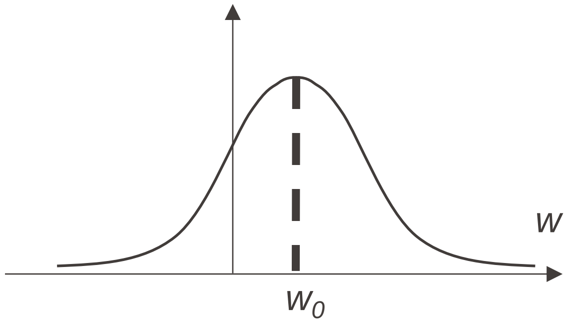
- Velocity distribution function
- Relationship between macroscopic properties and velocity distribution
- Evaporation kinetics (Schrage equation)

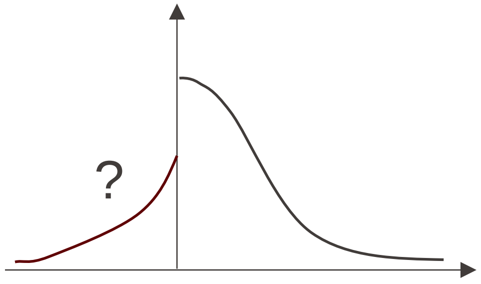
Mass Balance



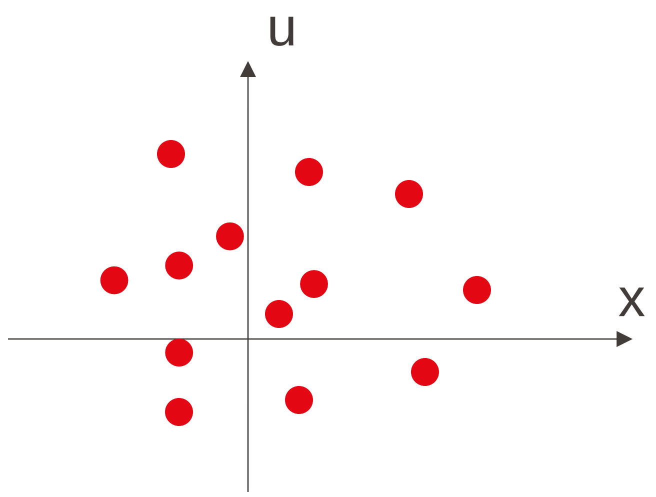
General Governing Equation







One-Dimensional Case



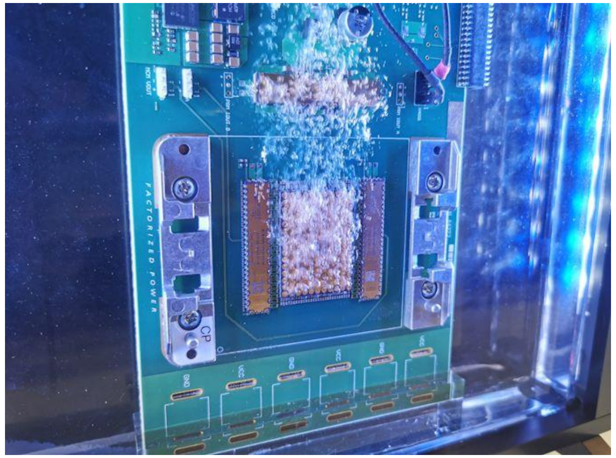
Boltzmann Equation

$$\frac{\partial F}{\partial t} = \frac{\partial F}{\partial x} \cdot u + \frac{\partial F}{\partial y} \cdot v + \frac{\partial F}{\partial y} \cdot z$$
$$+ \frac{\partial F}{\partial u} \cdot g_x + \frac{\partial F}{\partial v} \cdot g_y + \frac{\partial F}{\partial w} \cdot g_z + \Omega(F, F)$$

Boiling

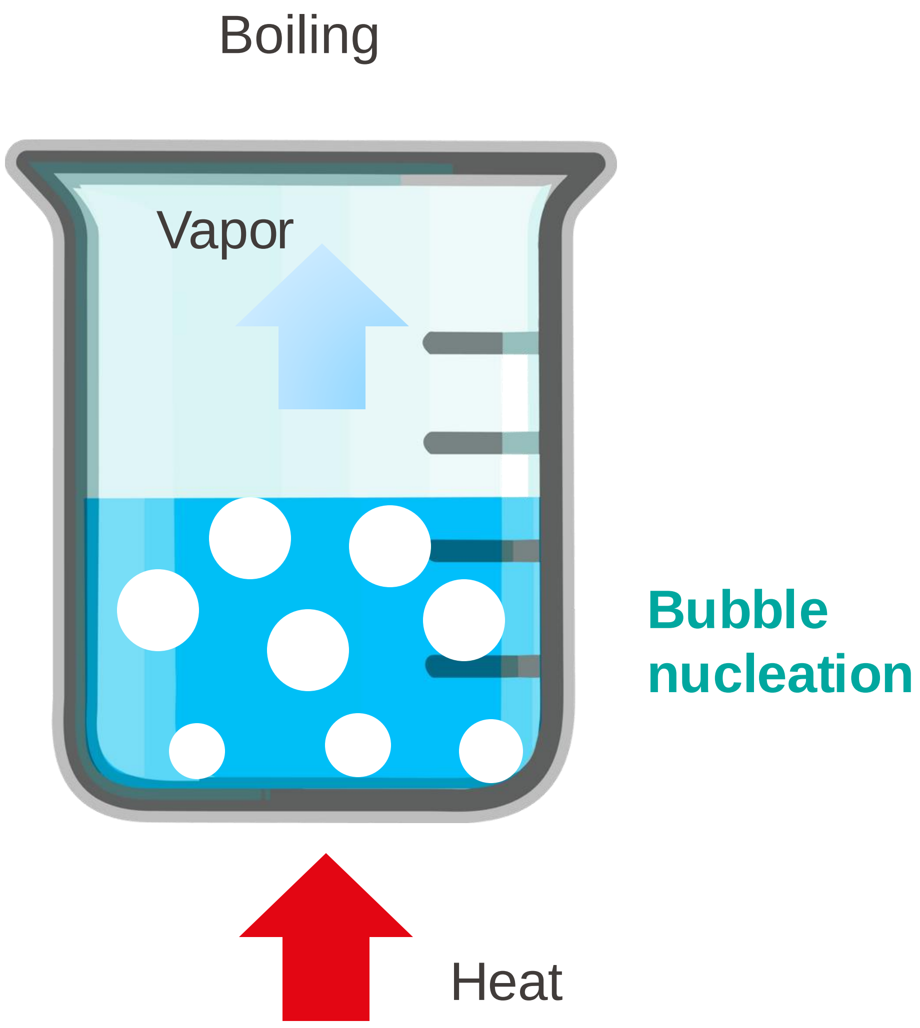
cooking

(nuclear) power plant



immersion cooling

Evaporation vs Boiling



Intended Learning Objectives Today

- Analyze the free energy of vapor embryo (Thermodynamics)
- Understand the derivation of bubble growth kinetics at small sizes

  - Reading materials: **Carey**, Chapter 5.2, 5.3, 6.1

Formation of a Vapor Embryo (Homogeneous Nucleation)

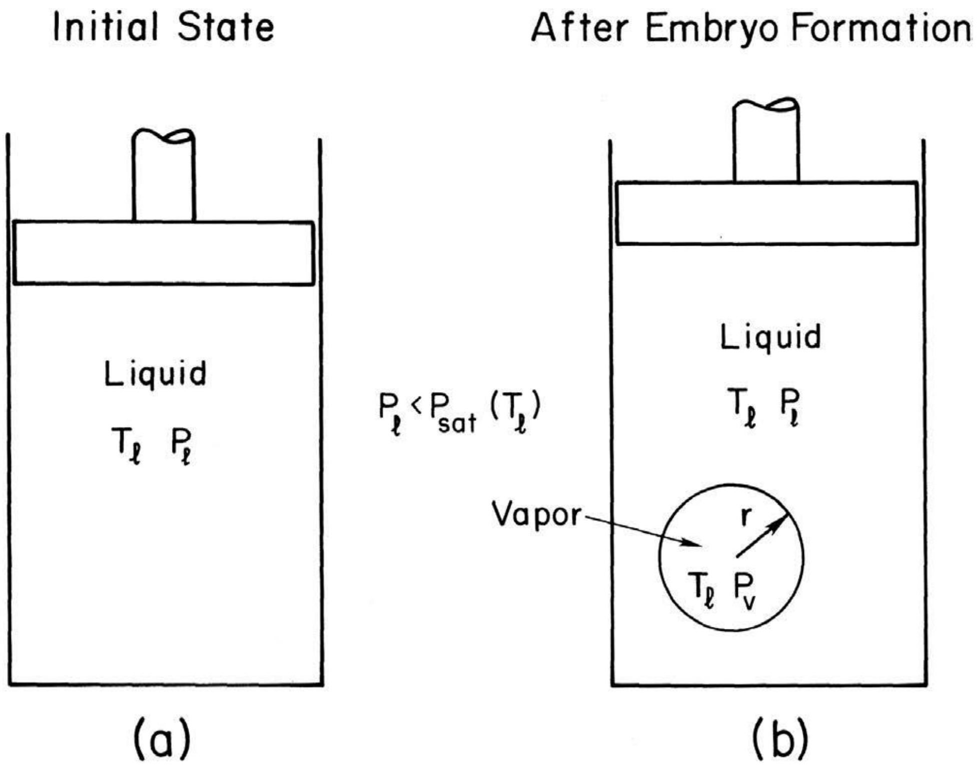
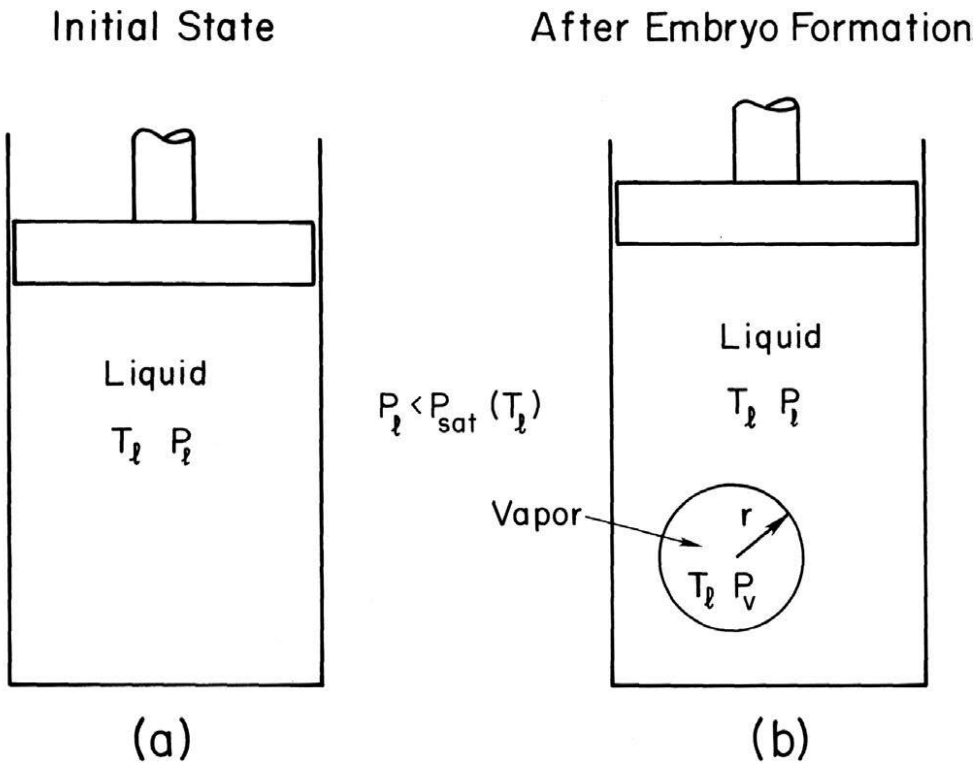


Figure 5.7 in Carey

Homogeneous Nucleation



Gibbs Free Energy Change

$$\Delta G = \widehat{N}_v(\hat{g}_v - \hat{g}_l) + (P_l - P_v)V_v + 4\pi r^2 \sigma_{lv}$$

Considering T_l and P_l as fixed values, assuming mechanical equilibrium is always satisfied

With ideal gas law, $\widehat{N}_v = P_v V_v / k_B T_l$ is also a function of r

ΔG can be considered as a function of r, to which we can apply Taylor expansion near r_e

Gibbs Free Energy Change

$$\frac{d\Delta G}{dr} = \frac{d\widehat{N}_v}{dr}(\hat{g}_v - \hat{g}_l) + 4\pi r^2\left(\frac{2\sigma_{lv}}{r} + P_l - P_v\right)$$

At $r = r_e$, $$\frac{d^2\Delta G}{dr^2} = -\frac{8\pi\sigma_{lv}}{3}\left(2 + \frac{1}{1 + \frac{2\sigma_{lv}}{r_e P_l}}\right)$$ Homework

Gibbs Free Energy Change

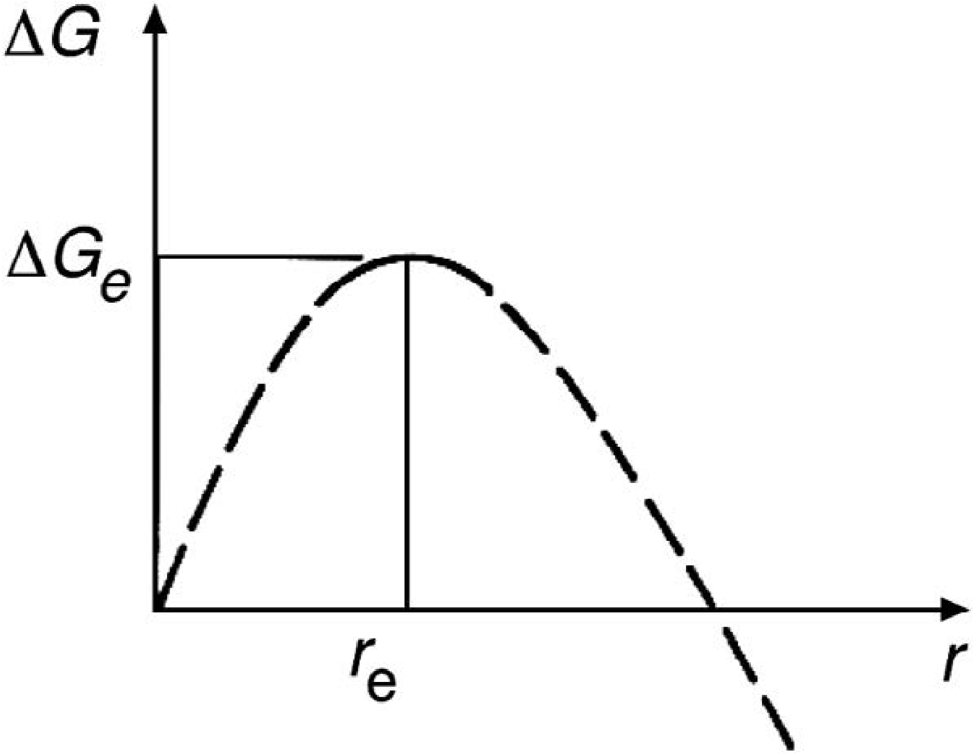


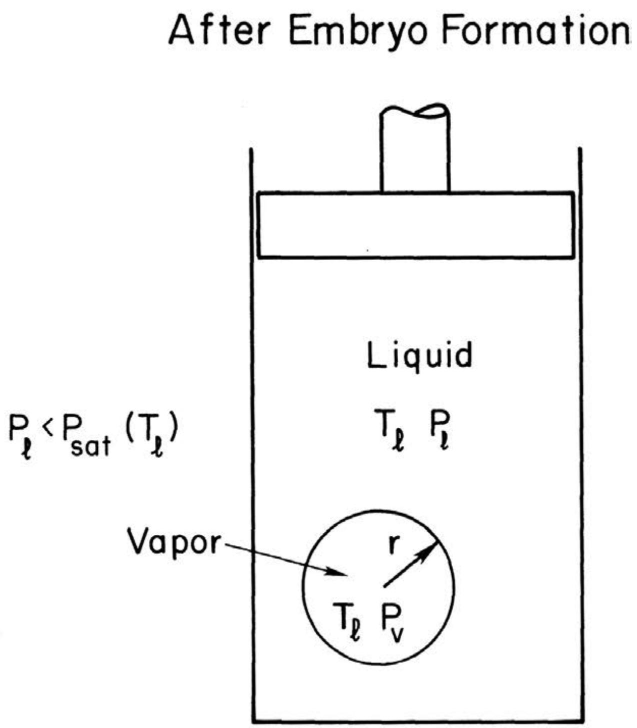
Figure 5.9 in Carey

Density fluctuation in superheated liquid produces bubbles of random r

If $r < r_e$, the bubble collapses
If $r > r_e$, the bubble grows

How do we determine r_e

Equilibrium Bubble Radius



Heterogeneous Nucleation

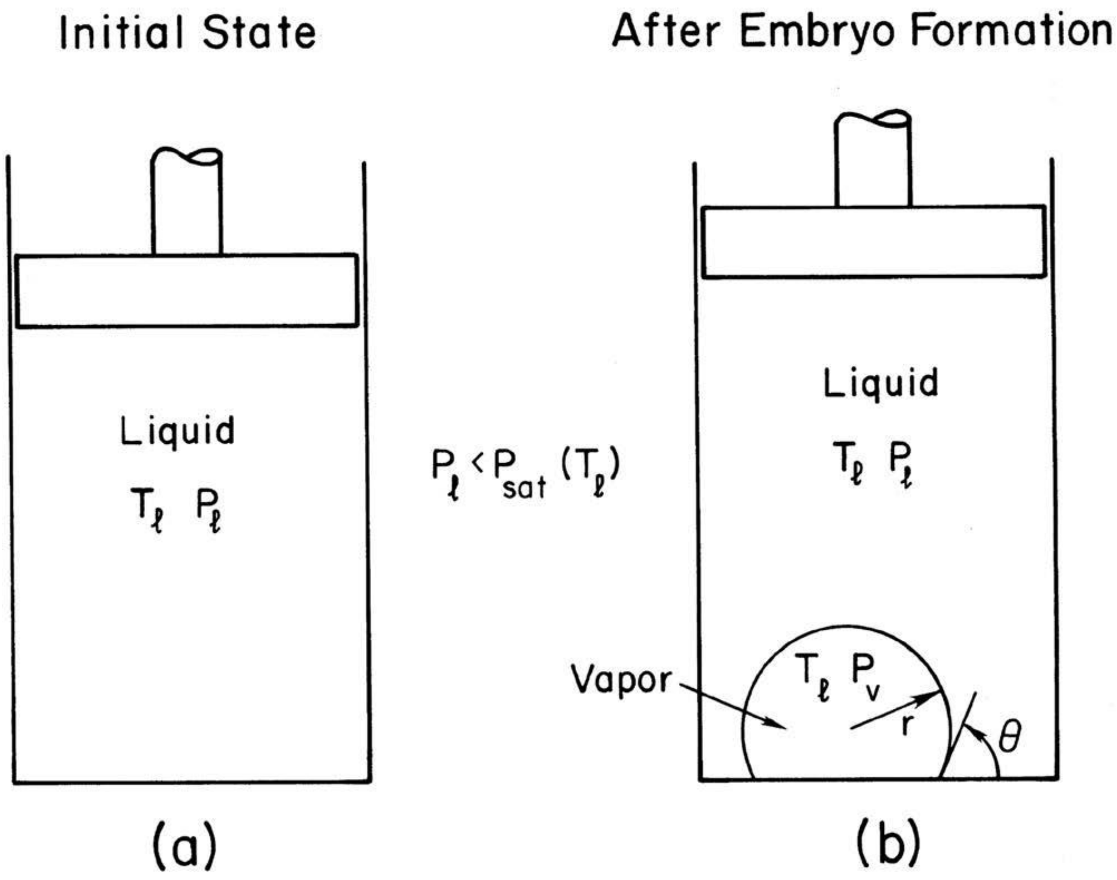
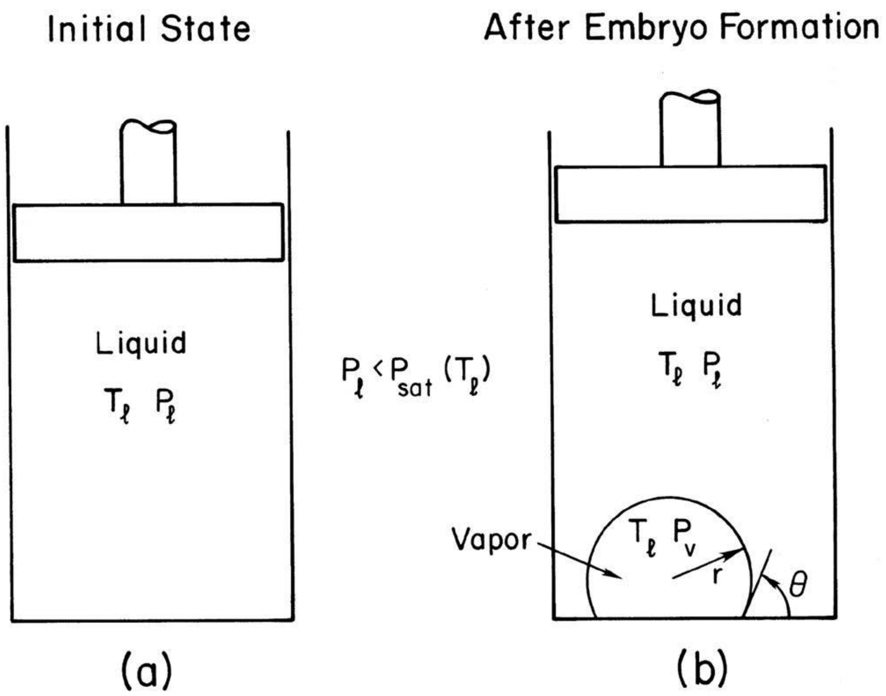
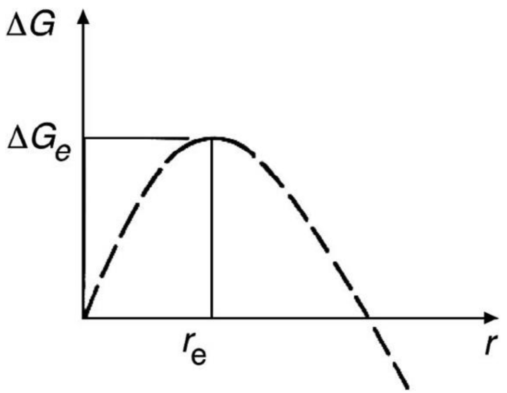


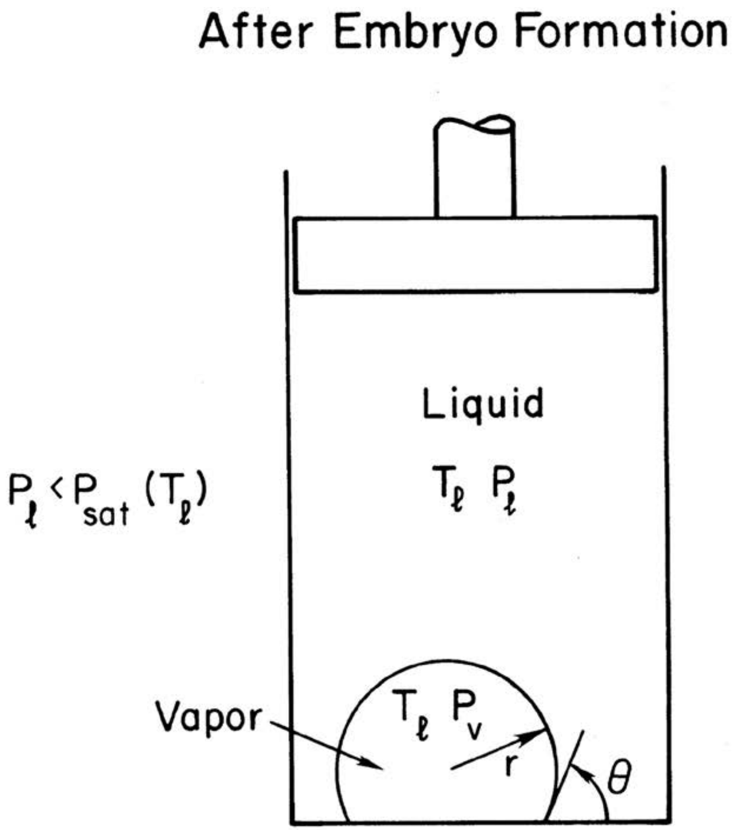
Figure 6.2 in Carey

Gibbs Free Energy Change





Equilibrium Bubble Radius



$$g_{sat,l}(T_l, P_{sat}) = g_{sat,v}(T_l, P_{sat}) = g_{sat}$$

$$dg = vdP - SdT$$

$$g_v - g_{sat} = \int_{P_{sat}}^{P_v} v_v dP = \int_{P_{sat}}^{P_v} \frac{RT_l}{P} dP = RT_l \ln\left(\frac{P_v}{P_{sat}}\right)$$

$$g_l - g_{sat} = \int_{P_{sat}}^{P_l} v_l dP = v_l(P_l - P_{sat})$$

In equilibrium $g_v = g_l$

$$\Rightarrow P_{ve} = P_{sat} \exp\left[\frac{v_l(P_l - P_{sat})}{RT_l}\right]$$

$$r_e = \frac{2\sigma}{P_{ve} - P_l} = \frac{2\sigma}{P_{sat} \exp\left[\frac{v_l(P_l - P_{sat})}{RT_l}\right] - P_l}$$

Gibbs Free Energy Barrier

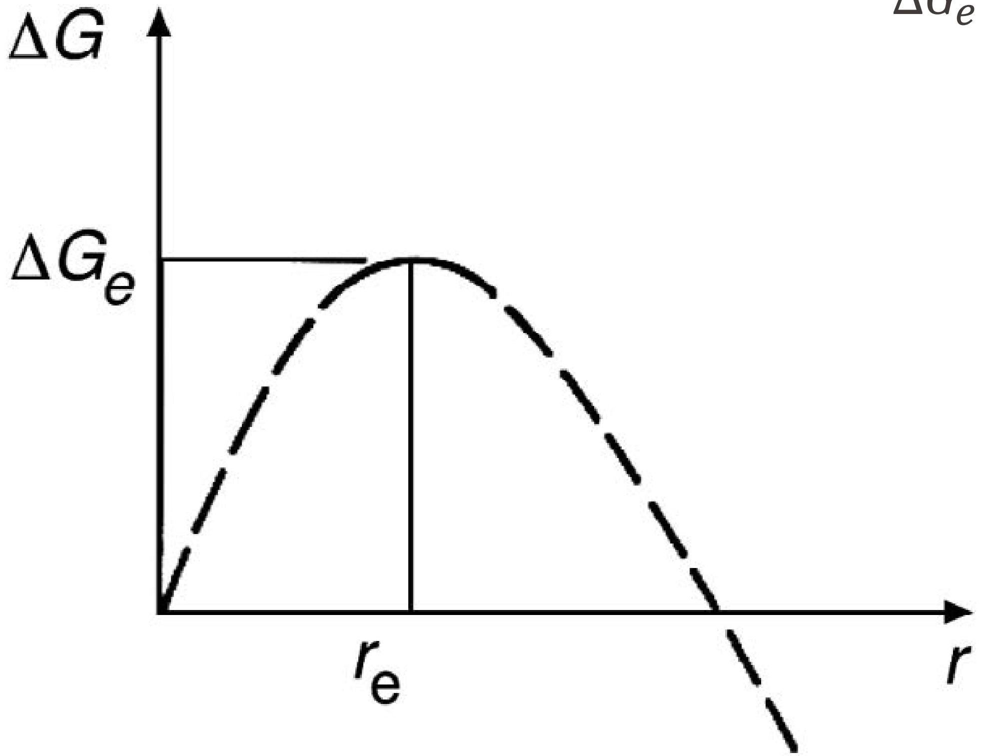


Figure 5.9 in Carey

$$\Delta G_e = \frac{4}{3}\pi r_e^2 \sigma_{lv}\left[\frac{1}{2} + \frac{3}{4}\cos\theta - \frac{1}{4}\cos^3\theta\right] = \frac{4}{3}\pi r_e^2 \sigma_{lv} F(\theta)$$

Embryo Size Distribution

Let's assume the number of embryos consisting of n molecules per unit volume N_n follows

$$N_n = \rho_{N,l} \exp\left[-\frac{\Delta G(r)}{k_B T_l}\right]$$

$\rho_{N,L}$ can be understood as the number of liquid molecules per unit volume ($\Delta G = 0$ corresponds to the liquid phase)

For an embryo of size n, define j_{ne} as the evaporating molecular flux and j_{nc} as the condensing molecular flux [$m^{-2}s^{-1}$]

Embryo Size Distribution

$$N_n A_n j_{ne} = N_{n+1} A_{n+1} j_{(n+1)c}$$

The rate at which n molecule embryos → n+1 molecule embryos through evaporation is the same as n+1 molecule embryos → n molecule embryos through condensation
No net exchange between two size groups

In superheated liquid, equilibrium is not necessarily satisfied

Consider the excess rate of n molecule embryos → n+1 molecule

Embryo Size Distribution

Embryo Size Distribution

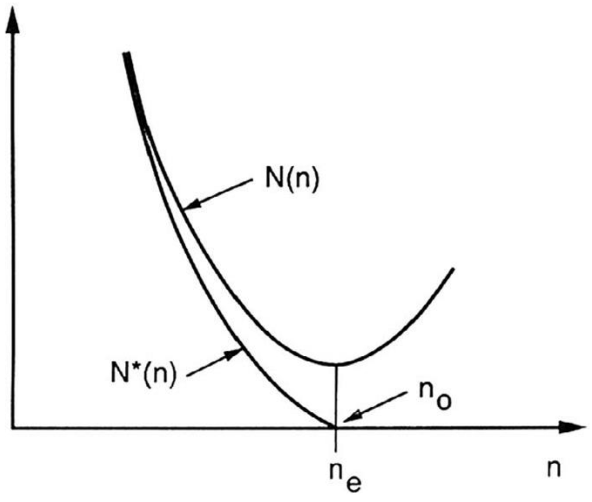
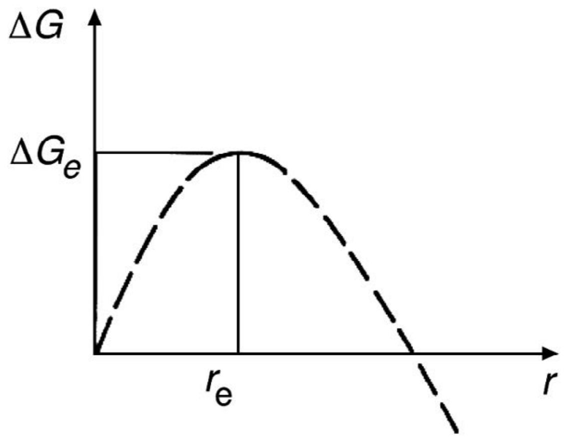


FIGURE 5.10 Carey

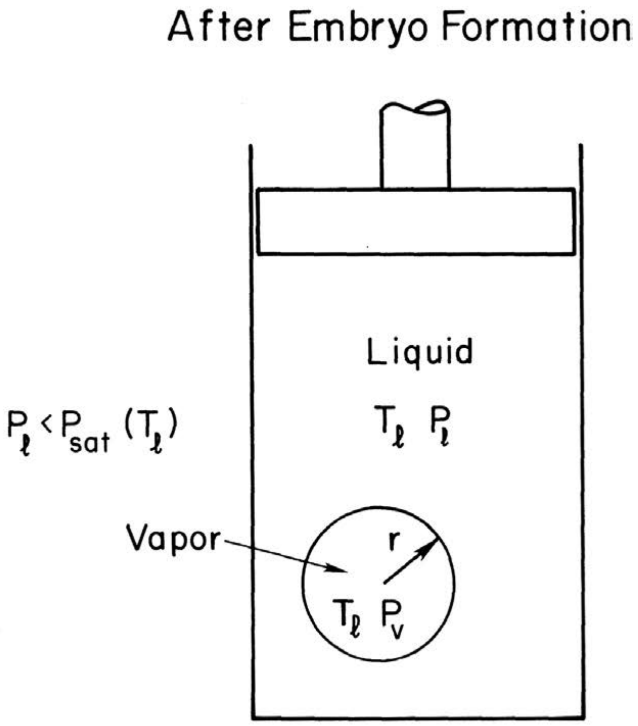
Embryo Size Distribution

Embryo Size Distribution



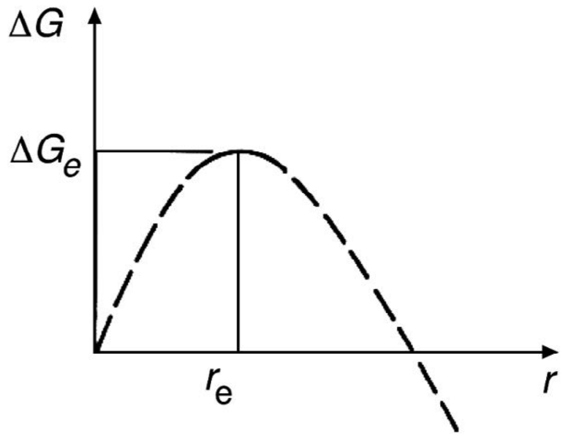
Embryo Size Distribution

$$J \approx \frac{P_{ve}}{\sqrt{2\pi m k_B T_l}} \left(\int_0^\infty [N(n)A(n)]^{-1}\, dn \right)^{-1}$$



Embryo Size Distribution

$$J \approx \frac{3\rho_{N,l}}{2 - P_l/P_{ve}} \left(\frac{k_B T_l}{2\pi m}\right)^{1/2} \left(\int_0^\infty \exp\left[\frac{\Delta G(r)}{k_B T_l}\right] dr\right)^{-1}$$



Physical Meaning of *J*

J represents the rate at which embryo bubbles grow from n to n + 1 molecules per unit volume [$m^{-3}s^{-1}$]

This includes the rate at which bubbles of the critical size are generated

Higher J implies higher probability of nucleation

Physical Meaning of *J*

$$J = \rho_{N,l}\left[\frac{6\sigma_{lv}}{\pi m\left(2-\frac{P_l}{P_{ve}}\right)}\right]^{1/2}\exp\left(-\frac{4\pi r_e^2\sigma_{lv}}{3k_B T_l}\right)$$ increases sharply with temperature

A change of 1°C can change *J* by as much as three or four orders of magnitude

We expect that there will exist a narrow range of temperature below which homogeneous nucleation does not occur, and above which it occurs almost immediately.

Generation Rate of Bubble of Critical Size

(Homogeneous case)

$$J = \rho_{N,l}\left[\frac{6\sigma_{lv}}{\pi m\left(2-\frac{P_l}{P_{ve}}\right)}\right]^{1/2}\exp\left(-\frac{4\pi r_e^2\sigma_{lv}}{3k_BT_l}\right)$$

increases sharply with temperature

There exists narrow range of temperature below which nucleation does not occur, and above which it occurs almost immediately.

$$10^{12}\ \mathrm{m}^{-3}\mathrm{s}^{-1} = 10^{-6}\mu\mathrm{m}^{-3}\mathrm{s}^{-1}$$

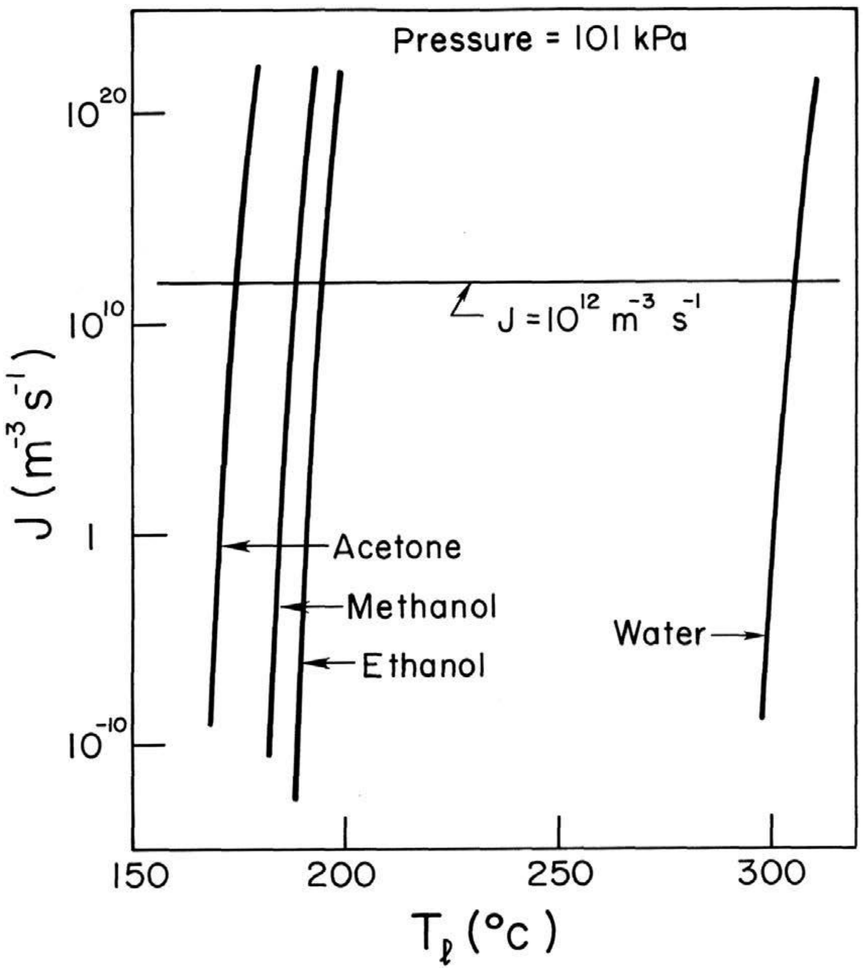


FIGURE 5.12, Carey

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